

Who has the power? An introduction to weighted voting

In many cases, where voting is used as a method of making policy, different voters have more votes than others. Examples of weighted voting occur in corporations where stockholders vote on issues, in the UN security council, and many other situations. An interesting question arises when we try to analyze how much power different voters have depending on the percent of the votes that they control. John Banzhaff became interested in this question and has developed a system to determine power.

The Banzhaff Power Index:

The technique that Banzhaff suggests involves trying to determine how many winning coalitions there are, and then to determine how often a particular voter makes a difference in that winning coalition. The algorithm is as follows.

- 1) List all coalitions of voters that are winning coalitions.
- 2) Select any voter and start that voter's power at 0.
- 3) From the list in step 1 select any coalition of which the voter is a member. Subtract from the coalition's vote total the number of votes that voter has. If the result is less than the number of votes necessary to pass an issue, add 1 to the voter's power count.
- 4) Repeat step 3 until you have checked all the coalitions for which that voter is a member.
- 5) Repeat steps 2 through 4 until you have checked all voters.
- 6) To find each voters power index, divide their power count by the total number of critical voters.

Example 1: Among the most important decisions a professional basketball team must make is the drafting of college players. In many cases the decision is made by a group of club officials, each with different votes. Take for example the Washington Wizards. In their system, the head coach (C) has 4 votes, the general manager (M) has 3 votes, the director of scouting operations (S) has two votes and the team psychiatrist (P) has 1 vote. Of the 10 votes cast, a simple majority of 6 votes is required for a yes vote on a player to be drafted.

{6: C, M, S, P}

This system can be represented symbolically by {6: 4, 3, 2, 1}, where 6 is the number needed to pass an issue and the other numbers are the votes of the different voters.

List all the minimal winning coalitions. Then determine the Banzhaff power index of each voter.

Example 2: Consider the weighted voting situation in which voters A, B, C, and D have 16, 12, 5, and 3 votes, respectively, and 21 votes are needed to pass an issue.

a) List all the winning coalitions and determine the Banzhaf power index for each voter.

b) Determine both the percentage of the votes and the percentage of the power for each voter.

	Percentage of votes	Percentage of power
A		
B		
C		
D		

c) Will each voter think that he or she has a fair share of the power? If not, who received more and who received less? Explain

Example 3: Consider a weighted voting system with three voters with 5, 4, and 1 vote respectively. If a simple majority is the quota, what is the quota for this system and write the system using bracket notation.

List all the winning coalitions and determine the Banzhaf power index for each voter. Are there any special names for any of the voters in this system?

c) Suppose a quota of 5 is used. What problems could arise?

d) Suppose a quota of 7 is used. How does the power change?

Example 4: Suppose a company has 4 stockholders: Alison, Benjamin, Chase, and Evan. They own 30%, 25%, 23% and 22% of the stock respectively. A simple majority (51%) is needed to pass an issue.

a) List the winning coalitions and determine the Banzhaf power index of each voter.

b) Determine the proportion of power of each voter and compare that to their holdings.

c) Suppose that Alison convinces Evan to sell her 3% of his stock so that she has 33% and Evan has 19%. Determine the new winning coalitions and decide if this has changed the balance of power.

d) Suppose instead that Benjamin convinces Evan to sell him the 3% of his shares instead, making the distribution Alison 30%, Benjamin 28%, Chase 23%, and Evan 19%. Determine how the power has changed with this distribution.

Example 5: Consider each of the following weighted voting systems. In each case, determine the Banzhaf power index of each voter. Be sure to identify interesting voters.

a. {6: 4, 2, 1}

b. {6: 7, 3, 1}

c. {17: 13, 5, 2, 1}

d. {6: 4, 3, 2, 1}

e. {5: 3, 2, 1, 1, 1}

f. {15: 11, 5, 2, 1}

Who has the power? Another Weighted Voting System

The Shapely-Shubik Power Index:

The algorithm is as follows.

- 1) List all arrangements of voters.
- 2) In each arrangement, select the one voter that is the first to make the coalition a winning coalition.
- 3) To find a voters Shapely-Shubik power index, count the number of times that voter is critical and divide by the total number of critical voters.

Example 1: {6: 4, 3, 2,} Then determine the Shapely-Shubik power index of each voter.

Example 2: Consider the weighted voting situation in which voters A, B, C, and D have 16, 12, and 5 votes, respectively, and 17 votes are needed to pass an issue.

a) Determine the Shapely-Shubik power index for each voter.

b) Determine both the percentage of the votes and the percentage of the power for each voter.

Percentage of
votes

Percentage of
power

A

B

C

c) Will each voter think that he or she has a fair share of the power? If not, who received more and who received less? Explain

Example 3: Consider a weighted voting system with three voters with 5, 4, and 1 vote respectively. If a simple majority is the quota, what is the quota for this system and write the system using bracket notation.

Determine the Shapely-Shubik power index for each voter.

Suppose a quota of 5 is used.

Suppose a quota of 7 is used. How does the power change?

Example 4: Suppose a company has 3 stockholders: Alison, Benjamin, and Chase. They own 30%, 45%, and 25% of the stock respectively. A simple majority (51%) is needed to pass an issue.

Determine the Shapely-Shubik power index for each voter.

Determine the proportion of power of each voter and compare that to their holdings.

Example 5: Consider each of the following weighted voting systems. In each case, determine the Shapely-Shubik power index of each voter. Be sure to identify interesting voters.

a. {6: 4, 2, 1}

b. {6: 7, 3, 1}

c. {16: 13, 5, 2}

d. {5: 4, 3, 2}

e. {3: 2, 1, 1}

f. {10: 11, 5, 2}

Weighted Voting Practice

1. A small high school has 150 students. Because of unexpectedly high yield in admissions, the sophomore class is significantly larger than the junior and senior classes: the sophomore class has 70 members while the junior and senior classes each have 40 members. The school's student council is composed of a single representative from each class. Each of the three members is given a number of votes proportional to the size of the class represented. Accordingly, the sophomore class has 7 votes, and the junior and senior reps each have 4 votes. The passage of any issue that is before the council requires a simple majority. Determine the Banzaff power index of each representative.

Determine the Shapely-Shubik power index of each representative.

2. Weighted voting is commonly used to decide issues at meetings of corporate stockholders. Each member is given one vote for each share of stock held. A company has four stockholders: A, B, C, and D. They own 47%, 41%, 7%, and 5% of the stock, respectively, and more than 50% of the vote is needed to pass an issue. Determine the Banzaff power index of each stockholder.

Determine the Shapely-Shubik power index of each stockholder.

3. $\{7;7,3,3\}$ Determine the Banzaff power index of each voter.

Determine the Shapely-Shubik power index of each voter.

4. $\{6;5,3,2\}$ Determine the Banzaff power index of each voter.

Determine the Shapely-Shubik power index of each voter.

5. $\{30;10,10,10,9\}$ Determine the Banzaff power index of each voter.

6. $\{8;8,5,1\}$ Determine the Banzaff power index of each voter.

Determine the Shapely-Shubik power index of each voter.

7. $\{101;99,98,3\}$ Determine the Banzaff power index of each voter.

Determine the Shapely-Shubik power index of each voter.

8. The UN Security Council consists of 5 permanent representatives (US, England, Russia, France, and China) and 10 countries that serve 2 year terms. By the rules of the UN, to pass an issue, all 5 permanent members plus 4 non-permanent members must vote for a resolution for it to pass. What effect does this have on the power of the permanent members and on the ability of someone like President Bush to get a measure passed?

9. In 1958, The Treaty of Rome established the existence of a yes-no voting system called the European Economic Community (EEC). The voters in this system were 6 countries, France, Germany, Italy, Belgium, the Netherlands, and Luxembourg. France, Germany, and Italy were given 4 votes each, Belgium and the Netherlands were given 2 votes each, and Luxembourg was given one vote. Passage of an issue required 12 of the 17 votes. Determine the Banzhaf power index of each country.

10. Debut of the Banzhaf Index: To support his 1965 lawsuit against the Board of Supervisors of Nassau County (NY), John Banzhaf introduced a method of measuring voting power. The county is divided into six districts. The 115 votes on the Board of Supervisors were allocated in 1964 to the districts as follows: 31 for Hempstead A, 31 for Hempstead B, 28 for Oyster Bay, 21 for North Hempstead, 2 for Long Beach, and 2 for Glen Cove. Assuming that decisions were made on the basis of simple majority, analyze the distribution of power to see what caused the controversy.

9. A five-person discipline committee has been established to deal with student infractions of minor school rules. The board is chaired by a Dean (D) and consists of two members of the teaching faculty (M and S) and two students (B and G). The teachers and the students each have 1 vote each. The committee was established with the intent that the dean would have considerably less power than the teachers and students. Therefore, the dean casts his one vote only in the case of a tie; otherwise cases are decided by a simple majority. Analyze the power structure (Banzaff method only) with this board.

10. $\{8;6,5,1\}$ Determine the Banzaff power index of each voter.

Determine the Shapely-Shubik power index of each voter.

11. A law firm is owned by four partners. When making decisions, each partner has 1 vote and the majority rules. In the case of a tie, the tie is broken in favor of the senior most partner. (for example, in a tie if the senior partner votes for a decision, then it passes. If he votes against the decision, then it fails.) Analyze this power structure (using the Banzaff method).